DECIDABILITY OF INTERPRETABILITY

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Homogeneous structures : Model Theory meets Universal Algebra

December 2025

Equivalence rel. of ω-cat. structures (Living on ω)

first-order

coarser

inter-def. $\quad \operatorname{Aut}\mathbb{A} = \operatorname{Aut}\mathbb{B}$

bi-def. $\quad \operatorname{Aut}\mathbb{A} \cong_{conj} \operatorname{Aut}\mathbb{B}$

$:\Leftrightarrow \exists \phi \in \operatorname{Sym}(\omega)$
$\operatorname{Aut}\mathbb{B} = \phi(\operatorname{Aut}\mathbb{A})\phi^{-1}$

bi-int. $\quad \operatorname{Aut}\mathbb{A} \cong_{top} \operatorname{Aut}\mathbb{B}$

$\operatorname{Aut}\mathbb{A} \cong_{alg} \operatorname{Aut}\mathbb{B}$

Equivalence rel. of ω-cat. structures (living on ω)

finer $\longrightarrow$

coarser $\downarrow$

	first-order	existential pos.
inter-def.	$\mathrm{Aut}\,\mathbb{A} = \mathrm{Aut}\,\mathbb{B}$	$\mathrm{End}\,\mathbb{A} = \mathrm{End}\,\mathbb{B}$
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bi-int.	$\mathrm{Aut}\,\mathbb{A} \cong_{top} \mathrm{Aut}\,\mathbb{B}$	$\mathrm{End}\,\mathbb{A} \cong_{top} \mathrm{End}\,\mathbb{B}$
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Equivalence rel. of ω-cat. structures (Living on ω)

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	first-order	existential pos.	primitive pos.
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RECONSTRUCTION

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RECONSTRUCTION

top. reconstruction

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	action reconstruction		
bi-int.	$\mathrm{Aut}\,\mathbb{A} \cong_{top} \mathrm{Aut}\,\mathbb{B}$	$\mathrm{End}\,\mathbb{A} \cong_{top} \mathrm{End}\,\mathbb{B}$	$\mathrm{Pol}\,\mathbb{A} \cong_{top} \mathrm{Pol}\,\mathbb{B}$
	top. reconstruction		
	$\mathrm{Aut}\,\mathbb{A} \cong_{alg} \mathrm{Aut}\,\mathbb{B}$	$\mathrm{End}\,\mathbb{A} \cong_{alg} \mathrm{End}\,\mathbb{B}$	$\mathrm{Pol}\,\mathbb{A} \cong_{alg} \mathrm{End}\,\mathbb{B}$

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RECONSTRUCTION

action reconstruction

top. reconstruction

"Guide to top. reconstruction" (Marimon + Pinsker 2025)

Why PoL ?

Why Pol?

$\mathbb{A}$ rel. structure in finite signature

CSP$\mathbb{A}$ computational problem:

$$\boxed{\begin{array}{l}\text{INPUT: finite rel. structure } \mathbb{I} \\ \text{QUESTION: } \mathbb{I} \xrightarrow{\text{hom.}} \mathbb{A}\ ?\end{array}}$$

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- Conjecture: P vs. NP-c. dichotomy for fo-reducts of fin. bounded homogeneous str. (Bodirsky + Pinsker 2011)

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- complexity of CSP $\mathbb{A}$ captured by:

$$\mathrm{Pol}\,\mathbb{A} := \bigcup_{n\in\omega} \{\mathbb{A}^n \xrightarrow{\text{hom.}} \mathbb{A}\}$$

$\mathbb{A}$ finite: very tight correspondence:

tractability $\sim$ existence nontriv. poly.

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$\mathbb{A}$ ω-cat: • $\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B} \Rightarrow$ CSP $\mathbb{A} \sim_{\text{P-time}}$ CSP $\mathbb{B}$ (Bodirsky + Nešetřil)

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- even for $\cong_{\text{top}}$ (Bodirsky + Pinsker)

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• even for $\cong_{top}$ (Bodirsky + Pinsker)

$\Longrightarrow$ investigate complexity of $\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B}$ / $\mathrm{Pol}\,\mathbb{A} \cong_{top} \mathrm{Pol}\,\mathbb{B}$

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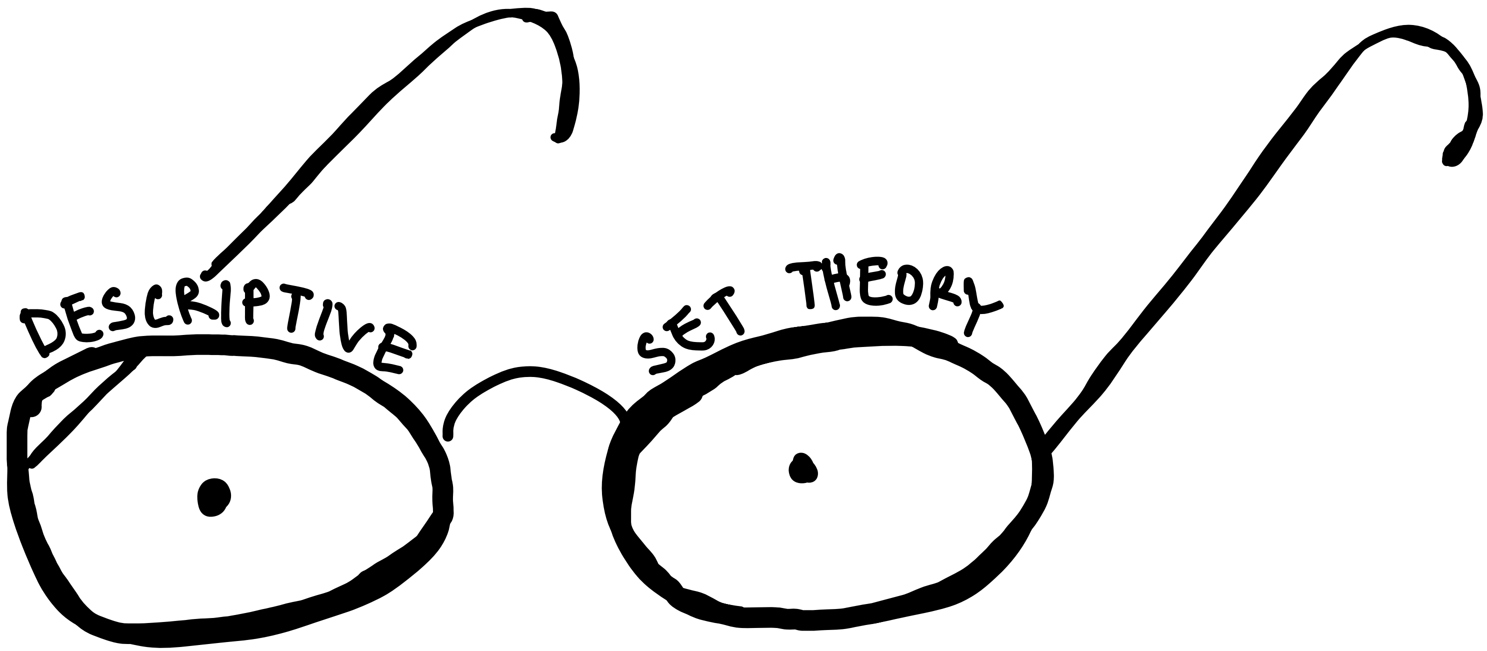

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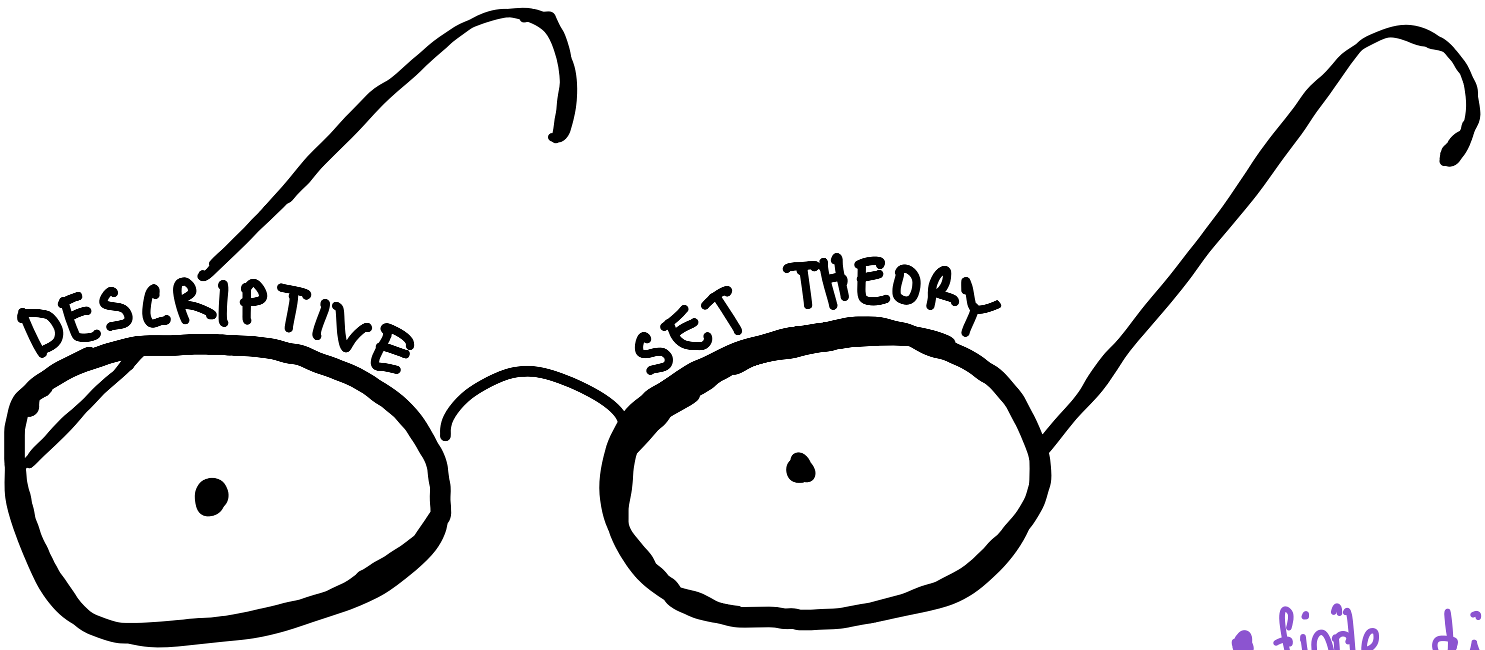

- finite discrete
- countable discrete
- $\mathbb{R}$

equiv. rel $\sim$ on X smooth $:\Leftrightarrow$ $\exists\, I : X \xrightarrow{\text{Borel}}$ std. Borel space

st. $x \sim y \Leftrightarrow I(x) = I(y)$.

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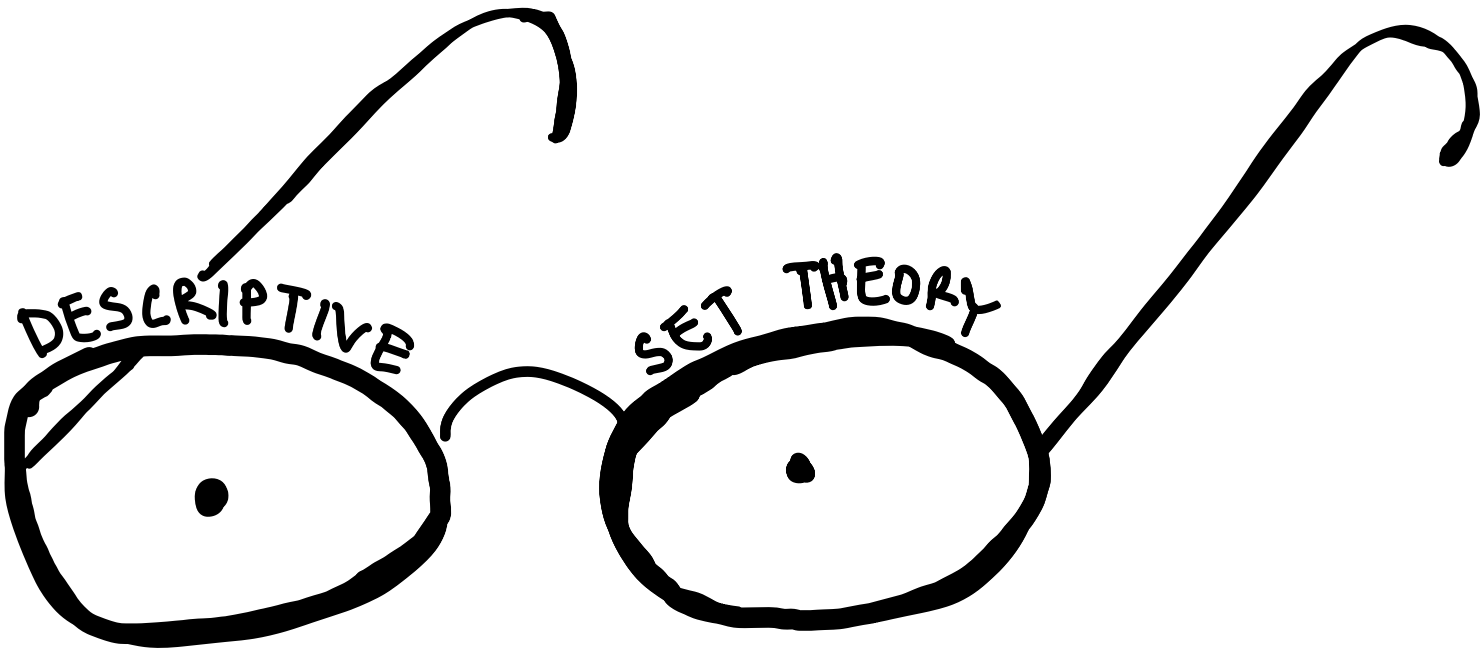

in "natural" way can assign x a "nice" invariant $I(x)$ (closed subgroups of $\mathrm{Sym}(\omega)$, fo-theory in fixed language, ...)

equiv. rel $\sim$ on X smooth $:\Leftrightarrow$ $\cancel{\exists I : X \xrightarrow{\text{Borel}} \text{std. Borel space}}$

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- assignment $I : \mathbb{A} \mapsto \mathrm{Th}(\mathcal{O}_{\mathrm{Aut}\,\mathbb{A}})$ is Borel
  - $\mathcal{O}_{\mathrm{Aut}\,\mathbb{A}} = (\omega ; (\sim_n)_{n\in\omega})$ $\quad x \sim_n y :\Leftrightarrow x, y \in \omega^n + \exists \alpha \in \mathrm{Aut}\,\mathbb{A}\ \alpha(x) = y.$
  - $\mathcal{O}_{\mathrm{Aut}\,\mathbb{A}} \cong \mathcal{O}_{\mathrm{Aut}\,\mathbb{B}} \Leftrightarrow \mathrm{Aut}\,\mathbb{A} \cong_{conj} \mathrm{Aut}\,\mathbb{B} + \mathcal{O}_{(\dots)}$ ω-cat.

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bi-int.	$\mathrm{Aut}\,\mathbb{A} \cong_{top} \mathrm{Aut}\,\mathbb{B}$	$\mathrm{End}\,\mathbb{A} \cong_{top} \mathrm{End}\,\mathbb{B}$	$\mathrm{Pol}\,\mathbb{A} \cong_{top} \mathrm{Pol}\,\mathbb{B}$	"pretty" smooth
	$\mathrm{Aut}\,\mathbb{A} \cong_{alg} \mathrm{Aut}\,\mathbb{B}$	$\mathrm{End}\,\mathbb{A} \cong_{alg} \mathrm{End}\,\mathbb{B}$	$\mathrm{Pol}\,\mathbb{A} \cong_{alg} \mathrm{End}\,\mathbb{B}$	

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(Nies + Paolini 2024) Smooth if restricted to NO ALGEBRAICITY / WEAK ELIMINATION OF IMAGINARIES

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- $\mathrm{End}\,\mathbb{A} \cong_{top} \mathrm{End}\,\mathbb{B} \overset{*}{\Rightarrow} \mathrm{End}\,\mathbb{A} \cong_{conj} \mathrm{End}\,\mathbb{B}$, $\mathrm{Pol}\,\mathbb{A} \cong_{top} \mathrm{Pol}\,\mathbb{B} \overset{*}{\Rightarrow} \mathrm{Pol}\,\mathbb{A} \cong_{conj} \mathrm{Pol}\,\mathbb{B}$

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(F. + Pinsker 2025/ongoing) Smooth if restricted to NO ALGEBRAICITY*

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inter-def.	$\mathrm{Aut}\,\mathbb{A} = \mathrm{Aut}\,\mathbb{B}$	$\mathrm{End}\,\mathbb{A} = \mathrm{End}\,\mathbb{B}$	$\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B}$	smooth
bi-def.	$\mathrm{Aut}\,\mathbb{A} \cong_{conj} \mathrm{Aut}\,\mathbb{B}$ $:\Leftrightarrow \exists \phi \in \mathrm{Sym}(\omega)$ $\mathrm{Aut}\,\mathbb{B} = \phi(\mathrm{Aut}\,\mathbb{A})\phi^{-1}$	$\mathrm{End}\,\mathbb{A} \cong_{conj} \mathrm{End}\,\mathbb{B}$	$\mathrm{Pol}\,\mathbb{A} \cong_{conj} \mathrm{Pol}\,\mathbb{B}$	smooth
bi-int.	$\mathrm{Aut}\,\mathbb{A} \cong_{top} \mathrm{Aut}\,\mathbb{B}$	$\mathrm{End}\,\mathbb{A} \cong_{top} \mathrm{End}\,\mathbb{B}$	$\mathrm{Pol}\,\mathbb{A} \cong_{top} \mathrm{Pol}\,\mathbb{B}$	"pretty" smooth
	$\mathrm{Aut}\,\mathbb{A} \cong_{alg} \mathrm{Aut}\,\mathbb{B}$	$\mathrm{End}\,\mathbb{A} \cong_{alg} \mathrm{End}\,\mathbb{B}$	$\mathrm{Pol}\,\mathbb{A} \cong_{alg} \mathrm{End}\,\mathbb{B}$	

Computational complexity?

Decidability of Definability

Decidability of Definability

- $\mathbb{C} = \mathrm{Flim}(\mathrm{Forb}(\underbrace{\mathcal{F}}_{\text{finite}}))$ (can list all n-orbits of $\mathbb{C}$)

Example: Random graph, $(\mathbb{Q}, <)$, K_n-free graph, ...

Decidability of Definability

- $\mathbb{C} = \mathrm{Flim}(\mathrm{Forb}(\underset{\text{finite}}{\mathcal{F}}))$ (can list all n-orbits of $\mathbb{C}$)
- fin. language fo-reducts $\mathbb{A}, \mathbb{B}$ of $\mathbb{C}$ (relations unions of $\mathbb{C}$-orb.)

Question:
- $\mathrm{Aut}\,\mathbb{A} = \mathrm{Aut}\,\mathbb{B}$?
- $\mathrm{End}\,\mathbb{A} = \mathrm{End}\,\mathbb{B}$?
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Too difficult!

Decidability of Definability

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Too difficult!

Add Ramsey

Decidability of Definability

- $\mathbb{C} = \mathrm{Flim}(\mathrm{Forb}(\mathcal{F}))$ Ramsey structure

  finite

- fin. language fo-reducts $\mathbb{A}, \mathbb{B}$ of $\mathbb{C}$

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(Bodirsky + Pinsker + Tsankov 2011)

For End, Pol this is decidable.

Decidability of Definability

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$\Leftrightarrow$ wlog $\exists f \in \mathrm{End}\,\mathbb{A} \setminus \mathrm{End}\,\mathbb{B}$

Decidability of Definability

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  finite
- fin. language fo-reducts $\mathbb{A}, \mathbb{B}$ of $\mathbb{C}$

Question:
- $\mathrm{Aut}\,\mathbb{A} = \mathrm{Aut}\,\mathbb{B}$?
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Decidability of Definability

- $\mathbb{C} = \text{Flim}(\text{Forb}(\mathcal{F}))$ — Ramsey structure

  finite

- fin. language fo-reducts $\mathbb{A}, \mathbb{B}$ of $\mathbb{C}$

Question:
- $\text{Aut}\,\mathbb{A} = \text{Aut}\,\mathbb{B}$? still open
- $\text{End}\,\mathbb{A} = \text{End}\,\mathbb{B}$?
- $\text{Pol}\,\mathbb{A} = \text{Pol}\,\mathbb{B}$?

(Bodirsky + Pinsker + Tsankov 2011)

For End, Pol this is decidable.

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Decidability of Interpretability

$\mathbb{A}$ model-complete core $:\Leftrightarrow$ $\overline{\mathrm{Aut}\,\mathbb{A}} = \mathrm{End}\,\mathbb{A}$

(Bodirsky 2005) $\mathbb{A}$ ω-cat. $\exists\, \mathbb{A}^{core}$ m-c core hom. equiv. to $\mathbb{A}$.
$\mathbb{A}^{core}$ unique up to isom. + ω-cat. $(\mathrm{CSP}\,\mathbb{A} = \mathrm{CSP}\,\mathbb{A}^{core})$

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Can compute $\mathbb{A}^{core}$.

(P. + Pinsker 2025) $\mathbb{A}, \mathbb{B}$ fo-reducts of $\mathrm{Flim}(\mathrm{Forb}\,\mathcal{F})$, $\mathrm{Flim}(\mathrm{Forb}\,\mathcal{G})$
Ramsey, without algebraicity.

Can decide:

- $\mathrm{End}\,\mathbb{A}^{core} \cong_{top} \mathrm{End}\,\mathbb{B}^{core}$
- $\mathrm{Pol}\,\mathbb{A}^{core} \cong_{top} \mathrm{Pol}\,\mathbb{B}^{core}$ *

Thank you!

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